P stabilizes dark matter and with CP can predict leptonic phases

Ravi Kuchimanchi*

(Dated: September 12, 2012)

We find that spontaneously broken parity (P) or left-right symmetry stabilizes dark matter in a beautiful way. Strong CP problem is solved by additionally imposing CP. Leptonic CP phases vanish at the tree level in the minimal strong CP solving model, which is a testable prediction. Experimentally if leptonic CP phases are not found (or are very small) it will be evidence for the type of models in this work where CP is spontaneously or softly broken and there is also a second hidden or softly broken symmetry such as P, Z_2 or Z_4. However leptonic CP violation can be present in closely related or non-minimal versions of these models.

PACS numbers: 11.30.Er, 95.35.+d, 14.60.Pq, 12.15.Ff

Introduction - Astronomical observations of galactic rotation curves [1] and velocity distribution of galaxies in clusters [2], smallness of anisotropies in the Cosmic Microwave Background radiation [3], and in a striking manner the Bullet Cluster [4], have all provided significant evidence that there is 4-5 times more matter in the universe that interacts gravitationally than is visible. The dominant thinking is that dark matter is comprised of a new non-baryonic particle, is electrically neutral and therefore almost transparent.

If dark matter is abundantly present it must be very stable. The standard approach to prevent it from decaying to normal matter is to introduce an unbroken Z_2 symmetry whereby the dark matter particles are odd under Z_2 while normal matter particles are even. This then implies that the *lightest* Z_2 odd particle is stable.

However introducing Z_2 only for the purpose of stability is unsatisfactory as it does not provide greater insight (see for example [5]). Moreover there is considerable arbitrariness in model building as there are many ways to introduce it. Therefore there is a need for a deeper approach to the problem of stability of dark matter.

In this work we first prove at a fundamental level that parity (P) can stabilize dark matter, though it is spontaneously broken. Illustrating this with left-right symmetric dark sector we show that Z_2 that is usually invoked to stabilize dark matter emerges as an automatic symmetry.

Since we require only P symmetry to stabilize dark matter we do not depend on stability ideas that use gauge symmetries such as $U(1)_{B-L}$ or $SO(10)$ that would also restrict us to multiplets with specific $B-L$ quantum numbers assigned so that R-parity or matter-parity are automatic symmetries [6, 7]. Our work is also distinct from models that require additional symmetries along with P to stabilize dark matter [8] or that look at viability of explaining dark matter issues in the minimal left-right model itself without additional fields [9].

P is well motivated not only on aesthetic grounds but also as is well known P requires that right-handed neutrinos must exist (and thus predicts that neutrinos have masses and can mix, as is now confirmed by observation), and as was shown in [10], P along with CP solves the strong CP problem. In [11] it was noted that leptonic CP phases can vanish in such models. In this work we show that even after inclusion of dark matter, leptonic CP phases vanish at tree level in the minimal strong CP solving model. We find more generally that CP with an additional symmetry (such as P, Z_2, or Z_4) can solve the strong CP problem and predict the absence of leptonic CP phases. If leptonic phases are not detected at the sensitivity of experiments such as in [12] that are being currently planned or underway, it will hint at CP being broken spontaneously (or softly) and there being a second hidden symmetry such as P, Z_2 or Z_4 as well in nature.

Parity stabilizes dark matter -

Claim 1. *Parity (P) can stabilize dark matter, even if it is spontaneously broken.*

Proof. Let P be a good symmetry of the Lagrangian (such as in the left-right symmetric model) and there be Higgs fields Δ_L and Δ_R that transform under P as $\Delta_L(x,t) \leftrightarrow \Delta_R(-x,t)$. Note that indices L and R on scalar fields are just labels. P is spontaneously broken (or hidden) when the neutral component Δ_R^o picks a constant vacuum expectation value (VEV) such that $\langle\Delta_R^o\rangle >> \langle\Delta_L^o\rangle$. However applying P twice it is easy to check that under P^2, $\Delta_L(x,t) \to \Delta_L(x,t)$, $\Delta_R(x,t) \to \Delta_R(x,t)$ and it follows that though P is broken, P^2 *remains unbroken* by these VEVs! Note that we have used the fact that since P is a good symmetry of the Lagrangian, so is P^2.

Classically P^2 (space inversion followed by space inversion) returns a system to its original state. But Quantum Mechanically this needs to be true only up to a phase. That is there can exist states ψ such that, $P^2\psi = e^{i\phi}\psi$ since only $|\psi|^2$ is physically observable and not the eigenstate ψ itself. Under P^2 different quantum fields can pick up different phases characterized by $e^{i\phi}$.

Note that $\eta \equiv \pm e^{i\phi/2}$ is called intrinsic parity as it is the parity of underlying P eigenstates [13]. Hence we use *intrinsic parity squared* for the P^2 eigenvalue $\eta^2 = e^{i\phi}$.

Since P^2 is conserved, the lightest particle χ with intrinsic parity squared $\eta_\chi^2 \neq 1$ (*we identify χ with dark*

Group	Q_{kL}	Q_{kR}	L_{kL} X_L, X'_R	L_{kR} X_R, X'_L	Δ_L	Δ_R	ϕ
$SU(3)_c$	3	3	1	1	1	1	1
$SU(2)_L$	2	1	2	1	3	1	2
$SU(2)_R$	1	2	1	2	1	3	2
$B-L$	1/3	1/3	-1	-1	2	2	0

TABLE I. Minimal left-right symmetric model with addition of dark sector X particles. In the above $k = 1$ to 3 corresponding to the usual 3 chiral families of quarks and leptons.

matter) cannot decay into a final state with intrinsic parity squared 1. That is into a final state consisting of particles that all have intrinsic parity squared 1 (or normal matter). This proves the stability of dark matter. □

Thus dark matter is matter with non-real intrinsic parity whose stability is elegantly understood to be a consequence of hidden parity symmetry and quantum nature of the universe. Parity does the job without need for symmetries with prefixes like R-parity and KK-parity [14].

Note that we can also use charge conjugation (C) instead of P in the above proof.

LR Symmetry and Dark Matter - We now consider the well known left-right symmetric group [15] $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$ with the minimal Higgs content needed for symmetry breaking, namely, an $SU(2)_R$ triplet Higgs field Δ_R that transforms as $(1, 1, 3, 2)$ (and its parity partner Δ_L), and the bidoublet ϕ that transforms as $(1, 2, 2, 0)$.

The matter content consists of the usual 3 generations of quarks and leptons represented by Q_{kL}, Q_{kR}, L_{kL} and L_{kR} (with $k = 1, 2, 3$) such that under P the gauge bosons of $SU(2)_L \leftrightarrow SU(2)_R$ and, $Q_{kL} \leftrightarrow Q_{kR},\ L_{kL} \leftrightarrow L_{kR},\ \Delta_L \leftrightarrow \Delta_R$ and $\phi \rightarrow \phi^\dagger$.

We add to this a fermionic particle X_L which is a doublet of $SU(2)_L$. Due to parity X_R which is a singlet of $SU(2)_L$ (and doublet of $SU(2)_R$), is automatically present. $X'_{R,L}$ are added to cancel chiral anomalies and along with $X_{L,R}$ they have the same gauge quantum numbers as the leptons (see Table I). We assign the 'X particles', X_L, X_R, X'_L and X'_R, with non-real intrinsic parity so that under P, $X_{L,R} \rightarrow \eta_X X_{R,L}$ and $X'_{L,R} \rightarrow \eta_X X'_{R,L}$ with $|\eta_X| = 1$ and $\eta_X^2 \neq 1$.

The most general parity symmetric Yukawa and mass terms of the Lagrangian involving the X particles are

$$L = \bar{X}_L \left(h\phi + \tilde{h}\tilde{\phi} \right) X_R + \bar{X}'_L \left(h'\phi^\dagger + \tilde{h}'\tilde{\phi}^\dagger \right) X'_R + \\ if(X_L^T C\tau_2 \Delta_L X_L + X_R^T C\tau_2 \Delta_R X_R) + if'(X_L'^T C\tau_2 \Delta_R X_L + \\ X_R'^T C\tau_2 \Delta_L X'_R) + M_X \bar{X}_L X'_R + M_X^\star \bar{X}'_L X_R + H.c. \quad (1)$$

Note that in the above, terms that could couple X particles to the usual leptons are automatically absent *due to parity*. This can be easily seen by applying parity twice (that is applying P^2) and using the P transformations mentioned above. For example under P^2 a term such as $\bar{L}_{kL}\phi X_R \rightarrow \eta_X^2 \bar{L}_{kL}\phi X_R$ and is not invariant for $\eta_X^2 \neq 1$ and is therefore absent.

Depending on the value of η_X two types of automatic symmetries follow as a consequence of P:

1. **Automatic $U(1)_D$ when $\eta_X^2 \neq \pm 1$**

   Note that under P^2 Majorana mass terms such as $fX_R^T C\tau_2 \Delta_R X_R \rightarrow \eta_X^4 fX_R^T C\tau_2 \Delta_R X_R$ and will be present only if $\eta_X^4 = 1$ or $\eta_X^2 = \pm 1$. If the phase η_X is chosen such that $\eta_X^2 \neq \pm 1$ then $f = f' = 0$ *due to parity* and the Majorana mass terms vanish. In this case we see that there is a residual automatic $U(1)_D$ in the model under which $X_{L,R} \rightarrow e^{i\beta} X_{L,R}$ and $X'_{L,R} \rightarrow e^{i\beta} X'_{L,R}$ for any real β. Since $U(1)_D$ is unbroken, the D-charge (or dark charge) is conserved. The number of X particles minus X antiparticles is then a constant.

2. **Automatic Z_2 when $\eta_X^2 = -1$**

   If $\eta_X = \pm i$ then the Majorana term (with $f, f' \neq 0$) is invariant under parity and is also present. In this case there is an automatic Z_2 symmetry in the model under which $X_{L,R}, X'_{L,R} \rightarrow -X_{L,R}, -X'_{L,R}$ and rest of the fields are unchanged under Z_2. Since P^2 eigenvalue $\eta_X^2 = -1$ for X-particles and is $+1$ for normal particles we in fact have $Z_2 \equiv P^2$.

Thus $U(1)_D$ or Z_2 symmetries usually imposed for stability are automatic consequences of P rather than being independently imposed and the lightest X particle is stable and identified with dark matter.

The mass matrix of the neutral X-fermions for the first case above is of the Dirac kind and is very simple: $\begin{pmatrix} h_o v & M_X \\ M_X^\star & h'_o v \end{pmatrix}$, where $h_o v$ can be written in terms of h, $\tilde{h}$ (similarly with primes for $h'_o v$) and the VEVs of ϕ (that determine the weak scale v).

The mass scale M_X is independent of the parity breaking scale set by $\langle \Delta_R^\circ \rangle$ and these scales can be anywhere between the weak scale and the Planck scale for both of the above cases.

The dark matter phenomenology may be interesting and needs to be explored in future. We also note that the assignment of non-real intrinsic parity to dark sector X particles can be done whether they are fermonic or bosonic and regardless of their gauge quantum numbers. Likewise to break $SU(2)_R$, instead of $B - L = 2$ triplet Higgs particles $(\Delta_{L,R})$ we can also use $B - L = 1$ doublet Higgses, since the dark sector's stability depends on P and not on $B - L$ assignments of particles in the model.

However the above model must be extended since it suffers from the strong CP problem. A way to do this is to invoke Peccei-Quinn symmetry [16] resulting in the well known axion as a dark matter candidate. However we

Group	Q_{iL}, Q'_R	Q_{iR}, Q'_L	L_{kL} X_L, X'_R	L_{kR} X_R, X'_L	Δ_L	Δ_R	ϕ
$SU(3)_c$	3	3	1	1	1	1	1
$SU(2)_L$	2	1	2	1	3	1	2
$SU(2)_R$	1	2	1	2	1	3	2
$B-L$	1/3	1/3	-1	-1	2	2	0

TABLE II. Matter Content of the minimal strong CP solving left-right symmetric model with fermonic dark matter candidate. In the above $k = 1$ to 3 corresponding to the usual 3 chiral families of leptons and $i = 1$ to 4 to include the 3 chiral families and fourth normal component of the vector like quark family

already have X-particles to serve as dark matter. Moreover the strong CP phase ($\bar{\theta}$) vanishes due to P itself, if it is unbroken. This provides strong motivation to resolve the strong CP problem without an axion as shown in [10] where P itself (with CP) ensures that $\bar{\theta}$ is not generated at the tree level even after spontaneous (or soft) breaking of P and CP. Moreover there is an experimentally testable prediction in this model where P stabilizes dark matter and with CP solves the strong CP problem – *the leptonic CP phases also vanish at the tree level.*

Absence of strong and leptonic CP - In order to solve the strong CP problem we impose both P and CP on the left-right symmetric Lagrangian and introduce a complete family of vectorlike quarks (with Q_{4L}, Q'_R making a vectorlike $SU(2)_L$ doublet and Q_{4R}, Q'_L making a vectorlike $SU(2)_R$ doublet) as in [10]. Along with the X-particles (that can be thought of as vector-like leptons), the fermionic and Higgs content of the model is as given in Table II .

However the vectorlike quarks and X-particles play different roles – while vectorlike quarks couple to the usual three light chiral families, and these interactions give rise to the CP violating phase in the CKM matrix of the quarks (when CP is softly or spontaneously broken) – the X particles on the other hand do not couple to the usual 3 families of leptons due to their intrinsic parity being non-real. They cannot both serve as dark matter as well as mix with normal matter to generate CP violation in the leptonic sector. Thus leptonic CP phases do not get generated.

We now show this more concretely. Under P we let $Q_{4L}, Q'_R \leftrightarrow Q_{4R}, Q'_L$. As in [10] CP is softly broken by dimension 3 terms, that are P symmetric namely

$$\sum_{i=1,4} M_i \bar{Q}_{iL} Q'_R + M_i^\star \bar{Q}'_L Q_{iR} + H.c. \tag{2}$$

Comparing the above with the last two terms of equation (1) we see that unlike $X'_{L,R}$, $Q'_{L,R}$ couple to all the quark families with complex CP violating couplings M_i. The mass of the vectorlike quarks $M \sim \sqrt{\sum |M_i|^2}$ can be any scale from just above the weak scale to the Planck scale while the complex phases in the ratios M_i/M generate the CKM CP phases when the vectorlike quarks decouple. Also, since the terms in (2) do not break P there is no $\bar{\theta}$ generated by them at the tree-level.

However we note that there is potentially another source for leptonic CP violation – complex VEVs for the bi-doublet ϕ which due to CP being imposed on dimension 4 terms, has real Yukawa couplings with quarks and leptons. Likewise if VEVs of triplets $\Delta_{L,R}$ that have real Yukawa couplings with the Leptons have relative complex phases, then they can generate leptonic CP violation along with Majorana masses and leptonic mixing. But any complex VEVs would break both P and CP and give rise to $\bar{\theta}$. Hence resolution of the strong CP problem and absence of leptonic CP phases are both linked to all Higgs VEVs being real.

Though CP is broken softly by dimension 3 terms in (2), the only possible soft CP breaking term in the scalar potential with the Higgs content in Table II is $\mu^2 Tr\tilde{\phi}^\dagger \phi + Hc.$ where $\tilde{\phi} = \tau_2 \phi^\star \tau_2$. Since P is a good symmetry, and keeping in mind that under $P, \phi \to \phi^\dagger$ it is easy to check that μ^2 is real due to P. This implies that all the Higgs VEVs that minimize the potential can be real thus solving the strong CP problem [10] and predicting the absence of leptonic CP violation at the tree-level...that is the absence of tree-level Dirac neutrino phase and Majorana phases in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.

Also, as shown in [10] the up and down quark mass matrices M_u and M_d are Hermitian since P is neither broken by terms in equation (2), nor by dimension 4 Yukawa terms and bi-doublet VEVs. Due to Hermiticity the tree-level strong CP phase $\bar{\theta} = \arg\det(M_u M_d) = 0$.

Non-minimal models - If we introduce an additional vectorlike lepton family that couples to the usual families, then it is easy to see that CP phases will be generated in the leptonic sector. The same thing happens if we make the dark X-particles bosonic and introduce one vector-like fermionic family that couples to all leptons. These are essentially extensions in the fermionic sector that go beyond what is minimally needed for dark matter and they give rise to leptonic CP violation.

On the other hand adding more Higgs fields beyond those in the minimal model, does not introduce leptonic CP violation. For example, a second Higgs bi-doublet and/or a singlet scalar can be added to break CP spontaneously instead of softly, as shown in reference [11]. However as discussed in [11], even with more scalars, vector like quarks that couple to usual quark families are needed to generate the CKM phase, or else CP phases from VEVs can be rotated away (if the strong CP problem is to be solved). Without vector like leptons, these same VEVs do not generate leptonic CP phases either and they vanish as was noted in [11]. Adding X-particles as dark matter as we have done in this work, does not

change this result as they do not couple to the usual leptons. In fact not only are the phases absent in the $PMNS$ matrix but if CP is broken spontaneously using 2 bi-doublets as in [11], then M_X in equation (1) is also real and there is no CP violating phase in the dark sector either.

Prevailing view on leptonic CP violation - If CP violation is hard in nature as it is in the standard model (through dimension 4 Yukawa couplings), then all CP violating phases including all the leptonic phases are expected to be present at the tree level, since they would be generated during renormalization. However the prevailing view in the field seems to be more biased than this – that whether CP violation is hard *or not*, it is expected that leptonic CP phases are present.

For example a recent review paper by Branco et al [17] states:

> "From a theoretical point of view, the complex phase in the CKM matrix may arise from complex Yukawa couplings and/or from a relative CP-violating phase in the vacuum expectation values (VEV) of Higgs fields. *In either case,* one expects an entirely analogous mechanism to arise in the lepton sector, leading to leptonic CP violation (LCPV)."

We have italicized some words for emphasis.

In fact to our knowledge so far no models (other than those based on P and CP we considered) have been studied where symmetries are responsible for the absence of leptonic CP violation, while a CP phase in the CKM matrix in the quark sector, and all the mixing angles of the neutrino sector including θ_{13}, are permitted or generated.

Our prediction of absence of tree-level leptonic CP violation differs from the prevalent view. We now consider more general ways of making this prediction.

Other symmetries - In lieu of P we now use other symmetries with CP and get vanishing leptonic CP violation by making slight changes to existing models:

1) A way to solve the strong CP problem using the Nelson-Barr mechanism [18], and have neutrino mixing, is the model by Branco et al. [19] where a vector-like iso-singlet down quark ($D_{L,R}$), 3 right handed neutrinos and a Higgs singlet S have been added to the usual standard model. CP and Z_4 are imposed in [19] so that under Z_4, $D_{L,R}, S \rightarrow -D_{L,R}, -S$ and $L_{kL}, e_{kR}, \nu_{kR} \rightarrow iL_{kL}, ie_{kR}, i\nu_{kR}$. When S picks a complex CP violating VEV then owing to its Yukawa coupling $D_L(h_k S + h'_k S^\star)d_{kR}$ with the usual 3 right-handed iso-singlet quarks d_{kR}, complex phases enter the quark mass matrix and the CKM phase is generated. However S and $S^\star$ also have Z_4 invariant Majorana type Yukawa couplings with the right handed neutrinos such as $\nu_{kR}C(f_{kl}S + f'_{kl}S^\star)\nu_{lR}$ that induce leptonic CP violation. Thus Branco et al find a common origin for the leptonic and quark sector CP phases.

However we note that if we change the transformation properties to make all leptons invariant under Z_4, then S would not couple to the leptons and there is no CP violation generated in the leptonic sector, while the quark sector remains unaffected. The neutrino mixing happens as terms such as $m_{kl}\nu_{kR}C\nu_{lR}$ are now permitted. m_{kL} and all leptonic Yukawa couplings are real due to CP and there is an absence of leptonic CP violation. Fermion sector (x) can be added and if $x \rightarrow ix$ under Z_4 they do not couple to leptons and can serve as stable dark matter.

In case of P that we considered earlier the matter content of the minimal model unambiguously predicts the absence of leptonic CP violation. However for the minimal matter content with Z_4, leptonic CP violation is present or absent depending on how Z_4 is imposed.

2) Even if we do not solve the strong CP problem, but break CP spontaneously by including a second Higgs doublet in the standard model (extended with 3 right handed neutrinos), we can impose a Z_2 symmetry so that the Higgs doublet (H_o which is Z_2 odd) that picks up the CP breaking VEV has Yukawa couplings that link one generation of quarks (Z_2 odd) with the remaining two generations of quarks (Z_2 even), while all the leptons (Z_2 even) couple only to the Higgs doublet (H_e which is Z_2 even) that has CP conserving VEV. CP and Z_2 are softly broken by dimension 2 terms. This model will once again lead to an absence of leptonic CP violation, but since both H_o and H_e have Yukawa couplings with the quarks the Jarlskog invariant is non-zero. However while several models with 2 Higgs doublets have been considered (for a review please see [20]), ones that lead to an absence of leptonic CP violation while generating the needed CKM Matrix and allowing all the neutrino mixing angles seem not to be studied so far.

Grand Unification - If there is grand unification, quarks and leptons will be in the same multiplets and leptonic CP phases would also be generally present.

However we began this work by using P to stabilize dark matter. It is hard to reconcile the stability of dark matter due to P, with a grand unified theory (GUT) group like $SO(10)$ (without imposing Z_2 by hand). On the other hand, in some $SO(10)$ models matter parity [7] or automatic R-parity [6] (rather than P) stabilizes dark matter without needing Z_2 to be imposed, but it is restrictive and does not for example work with the X particles we considered in equation (1). Thus the GUT feeling that there must be leptonic CP violation is not necessarily relevant to our approach.

Conclusion - We showed in this work that parity and quantum nature of the laws governing the universe may be at the heart of dark matter stability. Further P with CP solves the strong CP Problem and predicts the vanishing of tree-level leptonic phases (in the minimal model). If leptonic CP phases are not experimentally detected or are very small then it would imply that CP

is spontaneously or softly broken *and* there is also an additional hidden or softly broken symmetry in nature such as P, Z_2 or Z_4.

Basically if the Lagrangian of nature has CP symmetry then it must be violated softly and/or spontaneously to produce the CKM CP violating phase. If nature also has a second symmetry such as P, Z_2, or Z_4 then this can ensure that while CKM phase is generated the strong CP phase and leptonic CP phases do not also get generated at the tree level. The same symmetry (for example P) that protects the strong CP phase from getting generated can also protect the leptonic CP phases from being generated. This same symmetry can also stabilize dark matter.

Since leptonic CP violation can be generated radiatively, allowing at least a one-loop suppression, a conservative bound is that the induced leptonic phases are less than $\delta_{ckm}/(16\pi^2) \sim 0.5^\circ$ from the CP conserving values of 0 or π. Currently experiments are being planned or underway [12] to achieve a sensitivity of about 5°.

We also find that leptonic CP phases can be present in closely related or some non-minimal versions of these models as P, Z_2 or Z_4 can then be imposed so that only the strong CP phase is protected from being generated at the tree level.

* raviaravinda@gmail.com

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